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Calibration

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Analog to Digital (A/D) systems take real world analog values, such as temperature or pressure, and convert them to unitless digital values. They must be configured and calibrated to provide accurate real world engineering units for use by control programs.

Calibration

Analog to Digital (A/D) systems take real world analog values, such as temperature or pressure, and convert them to digital values so that your computer can observe them. These values can then be recorded or used to control a process. The digital values that are available to the computer are unitless integer numbers that are proportional to the real world values being read and must be converted into the real world values.

An A/D channel on a Rabbit BL2100, for instance, will return a value of 4095 to 0 that corresponds to an input voltage of -10.0 V to +10.0 V DC. These channels are attached to transducers which return voltages or currents that are linearly proportional to the real world values being observed.

To convert the unitless digital value into its corresponding real world value you use the equation of a straight line, Y = M * X + b, where X is the unitless digital value and Y is the corresponding real world value, usually a voltage (i.e. -10.0 V to +10.0 V) or a current (4.0 to 20.0 mA). M corresponds to the slope of the line and b corresponds to the Y intercept. When we say that the unitless value is linearly proportional to the real world value, we are referring to the slope component of this equation. As X increases, Y increases (or decreases) proportionally. The b component refers to the DC offset.

In order to perform the linear conversion, you need to find the correct M (slope) and b (Y intercept) values. Let's assume that we have a 12 bit A/D that returns values from 0 to 4095 (2^{12} - 1) when measuring values of 0.0 V to 10.0 V:

$$X_{lowAD} = 0,\ X_{highAD} = 4095,\ Y_{lowvoltage} = 0.0,\ Y_{highvoltage} = 10.0$$

$$M_{voltage} = (Y_{highvoltage} - Y_{lowvoltage})/(X_{highAD} - X_{lowAD})$$
$$M_{voltage} = (10.0 - 0.0)/(4095 - 0)$$
$$M_{voltage} = 0.002442$$

$$b_{voltage} = Y_{highvoltage} - (M_{voltage} * X_{highAD})$$
$$b_{voltage} = 10.0 - (0.002442 * 4095)$$
$$b_{voltage} = 0.00000181$$

$$Y_{voltage} = M_{voltage} * X_{AD} + b_{voltage}$$
$$Y_{voltage} = 0.002442 * X_{AD} + 0.00000181$$

$$X_{AD} = (Y_{voltage} - b_{voltage}) / M_{voltage}$$
$$X_{AD} = (Y_{voltage} - 0.00000181) / 0.002442$$

This $M_{voltage}$ and $b_{voltage}$ are the ideal values and assume a perfect conversion.

Lets also assume that we have pressure transducers that read pressures from 0 psia to 100 psia and return corresponding voltages of 0.0 V to 10.0 V. We can use this information to convert from the voltages computed above to real world engineering units:

$$X_{lowvoltage} = 0.0,\ X_{highvoltage} = 10.0,\ Y_{lowPSIA} = 0.0,\ Y_{highPSIA} = 100.0$$

$$M_{PSIA} = (Y_{highPSIA} - Y_{lowPSIA})/(X_{highvoltage} - X_{lowvoltage})$$
$$M_{PSIA} = (100.0 - 0.0)/(10.0 - 0.0)$$
$$M_{PSIA} = 10.0$$

$$B_{PSIA} = Y_{highPSIA} - (M_{PSIA} * X_{highvoltage})$$
$$B_{PSIA} = 100.0 - (10.0 * 10.0)$$
$$B_{PSIA} = 0.0$$

$$Y_{PSIA} = M_{PSIA} * X_{voltage} + B_{PSIA}$$
$$Y_{PSIA} = 10.0 * X_{AD} + 0.0$$

For efficiency reasons, we probably don't want to convert to voltage and then to PSIA, but rather directly to PSIA. We can now compute the values in PSIA that correspond to X_{lowAD} and X_{highAD}:

$$Y_{lowPSIA} = M_{PSIA} * X_{lowvoltage} + B_{PSIA}$$
$$Y_{lowPSIA} = 10.0 * 0.0 + 0.0$$
$$Y_{lowPSIA} = 0.0$$

$$Y_{highPSIA} = M_{PSIA} * X_{highvoltage} + B_{PSIA}$$
$$Y_{highPSIA} = 10.0 * 10.0 + 0.0$$
$$Y_{highPSIA} = 100.0$$

We can then use these values to convert directly from the A/D value to engineering units:

$$X_{lowAD} = 0,\ X_{highAD} = 4095,\ Y_{lowPSIA} = 0.0,\ Y_{highPSIA} = 100.0$$

$$M_{PSIA} = (Y_{highPSIA} - Y_{lowPSIA})/(X_{highAD} - X_{lowAD})$$
$$M_{PSIA} = (100.0 - 0.0)/(4095 - 0)$$
$$M_{PSIA} = 0.024420024$$

$$B_{PSIA} = Y_{highPSIA} - (M_{PSIA} * X_{highAD})$$
$$B_{PSIA} = 100.0 - (0.024420024 * 4095)$$
$$B_{PSIA} = 0.00000172$$

$$Y_{PSIA} = M_{PSIA} * X_{voltage} + B_{PSIA}$$
$$Y_{PSIA} = 0.024420024 * X_{AD} + 0.00000172$$
$$Y_{voltage} = 0.002442 * X_{AD} + 0.00000181$$

You can also do a more direct conversion by mathematically computing the new slope and intercept values:

if $y_1 = m_1 * x_1 + b_1$
and $y_2 = m_2 * x_2 + b_2$
then $y_2 = m_3 * x_1 + b_3$

Where:

$$m_3 = m_2 * m_1$$
$$b_3 = m_2 * b_1 + b_2$$

Proof:

$$y_2 = m_2 * (m_1 * x_1 + b_1) + b_2$$
$$y_2 = m_2 * m_1 * x_1 + m_2 * b_1 + b_2$$

$$y_2 = (m_2 * m_1) * x_1 + (m_2 * b_1 + b_2)$$

I have provided a couple of utilities on my webpage, http://www.the-control-freak.com, that make it a bit easier get these conversion factors. The first is called Cals.exe and it provides a dialog box based application to compute the m (slope) and b (Y intercept) values when you know x_{low}, x_{high}, y_{low} and y_{high}. The computed values are cut and paste accessible to make it easy to move them to your application. The second is called ReCal.exe and is used to compute the m_3 and b_3 values shown above.

ReCal.exe uses the reported (observed) and the corresponding calibrated (actual) values to recompute the corrected cal factors. This lets you initially use the datasheet information to compute the ideal calibration factors which will put you into the ballpark. Imperfect circuits and age related or environmental drift will cause you to want to recalibrate occasionally to achieve the most accurate readings possible.

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ReCal 1.0 (C) 2008 - 2011, David Cass Tyler, P.O. Box 1026, Willard, NM 87063

ReCal recomputes calibration factors based on observed inputs

Usage:  ReCal <M1> <b1> <X1> <Y1> <X2> <Y2>

Where:

M1 is the original slope (M)
b1 is the original intercept (b)
X1 is the first reported value
Y1 is the actual callibrated value corresponding to X1
X2 is the second reported value
Y2 is the actual callibrated value corresponding to X2

Example:

ReCal 26.1955 -119.815 0.0 0.0 39.3 38.8
M3 = 25.8622, b3 = -118.291

Where:

M3 is the corrected slope (M)
b3 is the corrected intercept (b)

The instrument will display calibrated values using the corrected cal factors
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ReCal.exe uses X_1 and X_2 to specify the values currently reported by the system and Y_1 and Y_2 to specify the actual calibrated engineering units. You want to measure inputs at approximately 10% and 90% of the full reading range. In the case of this example, we are measuring 0.0 PSIA at 0.0 VDC and 100.0 PSIA at 10.0 VDC, so we want to establish corrected values at approximately 10 PSIA and 90 PSIA. The reason that you do this is to avoid the "rails". Since the minimum value you can sense is 0.0 VDC, you can't tell if you are actually reading 0.0 VDC or -10.0 VDC. Likewise, a reading of 10.0 VDC might mean anything from 10.0 VDC to 100.0 VDC – you just can't tell. Thus, we measure close to the minimum and maximum values without actually encountering them.

Since atmospheric pressure is 14.7 PSIA at Sea Level, you would have to measure a partial vacuum to get a 10.0 PSIA reading so you might want to use the reading at atmospheric pressure instead since most of your readings will probably lie in the range of atmospheric pressure to 100.0 PSIA anyhow. If you normally don't exceed a value of 50.0 PSIA, you might want to take your upper reading there even though you have greater dynamic range than that.

We already discussed how to calibrate to the nominal datasheet values. You can use these values for your original slope and Y intercept (M_1 and b_1) values. You can then simultaneously take your 10% (X_1) and 90% (X_2) readings while measuring the real value with a trusted instrument (preferably a NIST traceable meter or other validated instrument) to find out what the readings should have been (Y_1 and Y_2 respectively). ReCal will then give you corrected slope and Y intercept values that will convert directly from A/D readings to real world engineering units.

A discussion of calibration should also include some mention of precision. National Instruments publishes an article, "Understanding Instrument Specifications -- How to Make Sense Out of the Jargon", that states that $Effective\ Number\ of\ Digits\ (ENOD) = log_{10}\ (range/resolution)$. The $range$ is defined as ($X_1 - X_2$) or 100.0 PSIA – 0.0 PSIA or 100.0 PSIA. Simplistically, the $resolution$ is defined as the $range/number\ of\ discreet\ steps$. The $number\ of\ discreet\ steps$ is defined as $2^{number\ of\ bits}$ so in our example above, the number of bits would be 12 so the $number\ of\ discreet\ steps$ would be 4096 and would be represented with values of 0 to 4095. This means that the $resolution$ would be 100.0/4096 or 0.024414063. Finally, the $ENOD = log_{10}(100.0/0.024414063)$ or 3.612359939 or effectively 3½ digits of precision. This implies that when you display the resulting measurement, you would only display it to 1 decimal place so the displayed values would be 0.0 PSIA, 0.1 PSIA, 0.2 PSIA … 100.0 PSIA. To display the number to a larger number of decimal places would imply a greater precision that just does not exist. You should also use an instrument that is NIST Traceable and that has a greater ENOD to perform your calibration.

Simplifying the above, we see that $ENOD=\log_{10}(R/(R/S))= \log_{10}(R*S/R)= \log_{10}(S)$ where S is $2^{(A/D\ bits)}$. Following is a table of the $Effective\ Number\ of\ Digits\ (ENOD)$ of the popular A/D converters:

A/D Bits	Discrete Steps	ENOD
8	256	2.408
12	4,096	3.612
16	65,536	4.816
24	16,777,216	7.225

32	4,294,967,296	9.633

We talked earlier about staying "off the rails" when doing A/D conversions. When you encounter these values ($X_{lowA/D}$ and $X_{highA/D}$) and, use them in a calculation, you are returning misleading and/or inaccurate values. IEEE Standard 754 for floating point numbers allows you to specify and recognize positive infinity and negative infinity. If you return negative infinity for $X_{lowA/D}$ and positive infinity for $X_{highA/D}$ you can recognize that you have "hit the rails" and determine which one you hit. If these numbers are used in a calculation, they propagate to the result so that you don't misrepresent a valid result. You can also represent missing or erroneous conversions as "not a numbers" and recognize them as well.

One last thing that I want to talk about is how versatile the equation of a straight line is – enough so that it has it's own assembly language instruction called a multiply-accumulate. You can convert from one scale to another, such as from distance in miles on a map to pixels on a screen ($Y_{pixels} = M_{pixels} * X_{miles} + b_{pixels}$). You can convert from one unit of measurement to another, such as from degrees Fahrenheit to degrees Centigrade ($Y_{centigrade} = M_{centigrade} * X_{fahrenheit} + b_{centegrade}$). You can convert from one currency to another, such as from British Pounds to US Dollars ($Y_{dollars} = M_{dollars} * X_{pounds} + b_{dollars}$). You can convert from units to profit, such as from couches to dollars ($Y_{dollars} = M_{dollars} * X_{couches} + b_{dollars}$). Heck, you can even convert from one type of measurement to another, such as hemoglobin A1C to average daily blood sugar ($Y_{AvgBloodSugar} = M_{AvgBloodSugar} * X_{A1C} + b_{AvgBloodSugar}$). The equations that were given earlier can be used to compute any or all of these conversions as long as Y varies proportionally to X.

There are all kinds of hardware considerations in performing good A/D conversions and they are documented elsewhere and are available on the web under the subject "signal preconditioning". There are also many texts on A/D conversions and the effects of frequency, bandwidth, aliasing and a host of other considerations. You can spend a career on this subject and its applications. Lord Kelvin, back in the 18th century, said "When you can measure what you are speaking about, and can express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, then your knowledge is of a meager and unsatisfactory kind". If your measurements are not precise (calibrated) you are, at best, just guessing.

Good luck on your quest for the perfect conversion.